Traduction d'un programme fouine en fouine CPS

On notera en cyan les variables fraîches. La continuation k est une variable fraîche mais qui reste constante tout au long de la transformation.

- $[\![n]\!] := \texttt{fun}\ k \to (\mathit{fst}\ k)\ n$
- $[\![b]\!] := \texttt{fun}\ k \to (\mathit{fst}\ k)\ b$
- $[\![\texttt{()}]\!] := \texttt{fun}\ k \to (\mathit{fst}\ k)\ \texttt{()}$
- $[\![x]\!] := \texttt{fun}\ k \to (\mathit{fst}\ k)\ x$
- $[\![\texttt{fun}\ x \to e]\!] := \texttt{fun}\ k \to (\mathit{fst}\ k)\ (\texttt{fun}\ x \to [\![e]\!])$
- $[\![e_1\ e_2]\!] := \texttt{fun}\ k \to [\![e_2]\!]\ (\texttt{fun}\ v \to [\![e_1]\!]\ (\texttt{fun}\ f \to f\ v\ k, \mathit{snd}\ k), \mathit{snd}\ k)$
- $[\![e_1\ \texttt{\&\&}\ e_2]\!] := [\![\texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else false}]\!]$
- $[\![e_1\ \texttt{||}\ e_2]\!] := [\![\texttt{if}\ e_1\ \texttt{then true else}\ e_2]\!]$
- $[\![e_1 \circledast e_2]\!] := \texttt{fun}\ k \to [\![e_2]\!]\ (\texttt{fun}\ v_2 \to [\![e_1]\!]\ (\texttt{fun}\ v_1 \to (\mathit{fst}\ k)\ (v_1 \circledast v_2), \mathit{snd}\ k), \mathit{snd}\ k)$
- $[\![\texttt{if}\ b\ \texttt{then}\ e_1\ \texttt{else}\ e_2]\!] := \texttt{fun}\ k \to [\![b]\!]\ (\texttt{fun}\ v \to \texttt{if}\ v\ \texttt{then}\ [\![e_1]\!]\ k\ \texttt{else}\ [\![e_2]\!]\ k, \mathit{snd}\ k)$
- $[\![\circledast\ e]\!] := \texttt{fun}\ k \to [\![e]\!]\ (\texttt{fun}\ v \to (\mathit{fst}\ k)\ (\circledast\ v), \mathit{snd}\ k)$
- $[\![e_1\ \texttt{;}\ e_2]\!] := \texttt{fun}\ k \to [\![e_1]\!]\ (\texttt{fun}\ \_ \to [\![e_2]\!]\ k, \mathit{snd}\ k)$
- $[\![\mathrm{C}(e_1, \dots, e_n)]\!] := \texttt{fun}\ k \to [\![e_n]\!]\ (\texttt{fun}\ v_n \to \dots\ ([\![e_1]\!]\ (\texttt{fun}\ v_1 \to (\mathit{fst}\ k)\ \mathrm{C}(v_1, \dots, v_n), \mathit{snd}\ k) \dots), \mathit{snd}\ k)$
- $$\begin{array}{l} [\![\texttt{while}\ b\ \texttt{do}\ e]\!] := \texttt{let rec}\ \mathit{boucle}\ k\ \texttt{=} \\ \qquad [\![b]\!]\ (\texttt{fun}\ v \to \\ \qquad\quad \texttt{if}\ v\ \texttt{then}\ [\![e]\!]\ (\texttt{fun}\ \_ \to \mathit{boucle}\ k, \mathit{snd}\ k) \\ \qquad\quad \texttt{else}\ (\mathit{fst}\ k)\ \texttt{()} \\ \quad \texttt{in}\ \mathit{boucle} \end{array}$$
- $$\begin{array}{l} [\![\texttt{for}\ i\ \texttt{=}\ e_1\ \texttt{to}\ e_2\ \texttt{do}\ e_3\ \texttt{done}]\!] := \texttt{fun}\ k \to [\![e_1]\!]\ (\texttt{fun}\ v_1 \to [\![e_2]\!]\ (\texttt{fun}\ v_2 \to \\ \qquad \texttt{let rec}\ \mathit{boucle}\ i\ k\ \texttt{=} \\ \qquad\quad \texttt{if}\ i \le v_2\ \texttt{then}\ [\![e_3]\!]\ (\texttt{fun}\ \_ \to \mathit{boucle}\ (i+1)\ k, \mathit{snd}\ k) \\ \qquad\quad \texttt{else}\ (\mathit{fst}\ k)\ \texttt{()} \\ \qquad \texttt{in}\ \mathit{boucle}\ v_1)) \end{array}$$
- $$\begin{array}{l} [\![\texttt{for}\ i\ \texttt{=}\ e_1\ \texttt{downto}\ e_2\ \texttt{do}\ e_3\ \texttt{done}]\!] := \texttt{fun}\ k \to [\![e_1]\!]\ (\texttt{fun}\ v_1 \to [\![e_2]\!]\ (\texttt{fun}\ v_2 \to \\ \qquad \texttt{let rec}\ \mathit{boucle}\ i\ k\ \texttt{=} \\ \qquad\quad \texttt{if}\ i \ge v_2\ \texttt{then}\ [\![e_3]\!]\ (\texttt{fun}\ \_ \to \mathit{boucle}\ (i-1)\ k, \mathit{snd}\ k) \\ \qquad\quad \texttt{else}\ (\mathit{fst}\ k)\ \texttt{()} \\ \qquad \texttt{in}\ \mathit{boucle}\ v_1)) \end{array}$$
- $$\begin{array}{l} [\![\texttt{match}\ e\ \texttt{with}\ p_1\ \texttt{when}\ e_1' \to e_1 \mid \cdots \mid p_n\ \texttt{when}\ e_n' \to e_n]\!] := \texttt{fun}\ k \to [\![e]\!]\ (\cdots(\texttt{fun}\ \mathit{match}_{\text{next}} \to \texttt{fun}\ v \to \\ \qquad \texttt{match}\ v\ \texttt{with} \\ \qquad \mid p_1 \to [\![e_1']\!]\ (\texttt{fun}\ v' \to \\ \qquad\qquad \texttt{if}\ v'\ \texttt{then}\ [\![e_1]\!]\ k \\ \qquad\qquad \texttt{else}\ \mathit{match}_{\text{next}}\ v, \\ \qquad\quad \mathit{snd}\ k) \\ \qquad \mid \_ \to \mathit{match}_{\text{next}}\ v, \\ \quad) \\ \quad (\texttt{fun}\ \mathit{match}_{\text{next}} \to \texttt{fun}\ v \to \\ \qquad \texttt{match}\ v\ \texttt{with} \\ \qquad \mid p_2 \to [\![e_2']\!]\ (\texttt{fun}\ v' \to \\ \qquad\qquad \texttt{if}\ v'\ \texttt{then}\ [\![e_2]\!]\ k \\ \qquad\qquad \texttt{else}\ \mathit{match}_{\text{next}}\ v, \\ \qquad\quad \mathit{snd}\ k) \\ \qquad \mid \_ \to \mathit{match}_{\text{next}}\ v, \\ \qquad) \\ \qquad\qquad\qquad \vdots \\ \quad (\texttt{fun}\ \mathit{match}_{\text{next}} \to \texttt{fun}\ v \to \\ \qquad \texttt{match}\ v\ \texttt{with} \\ \qquad \mid p_n \to [\![e_n']\!]\ (\texttt{fun}\ v' \to \\ \qquad\qquad \texttt{if}\ v'\ \texttt{then}\ [\![e_n]\!]\ k \\ \qquad\qquad \texttt{else}\ \mathit{match}_{\text{next}}\ v, \\ \qquad\quad \mathit{snd}\ k) \\ \qquad \mid \_ \to \mathit{match}_{\text{next}}\ v, \\ \quad) \\ \quad (\texttt{fun}\ \_ \to (\mathit{snd}\ k)\ \textbf{MatchError}))\cdots) \end{array}$$
- $$\begin{array}{l} [\![\texttt{try}\ e\ \texttt{with}\ p_1\ \texttt{when}\ e_1' \to e_1 \mid \cdots \mid p_n\ \texttt{when}\ e_n' \to e_n]\!] := \texttt{fun}\ k \to [\![e]\!]\ (\mathit{fst}\ k, \texttt{fun}\ v \to \\ \quad (\cdots(\texttt{fun}\ \mathit{match}_{\text{next}} \to \texttt{fun}\ v \to \\ \qquad \texttt{match}\ v\ \texttt{with} \\ \qquad \mid p_1 \to [\![e_1']\!]\ (\texttt{fun}\ v' \to \\ \qquad\qquad \texttt{if}\ v'\ \texttt{then}\ [\![e_1]\!]\ k \\ \qquad\qquad \texttt{else}\ \mathit{match}_{\text{next}}\ v, \\ \qquad\quad \mathit{snd}\ k) \\ \qquad \mid \_ \to \mathit{match}_{\text{next}}\ v, \\ \quad) \\ \quad (\texttt{fun}\ \mathit{match}_{\text{next}} \to \texttt{fun}\ v \to \\ \qquad \texttt{match}\ v\ \texttt{with} \\ \qquad \mid p_2 \to [\![e_2']\!]\ (\texttt{fun}\ v' \to \\ \qquad\qquad \texttt{if}\ v'\ \texttt{then}\ [\![e_2]\!]\ k \\ \qquad\qquad \texttt{else}\ \mathit{match}_{\text{next}}\ v, \\ \qquad\quad \mathit{snd}\ k) \\ \qquad \mid \_ \to \mathit{match}_{\text{next}}\ v, \\ \quad) \\ \qquad\qquad\qquad \vdots \\ \quad (\texttt{fun}\ \mathit{match}_{\text{next}} \to \texttt{fun}\ v \to \\ \qquad \texttt{match}\ v\ \texttt{with} \\ \qquad \mid p_n \to [\![e_n']\!]\ (\texttt{fun}\ v' \to \\ \qquad\qquad \texttt{if}\ v'\ \texttt{then}\ [\![e_n]\!]\ k \\ \qquad\qquad \texttt{else}\ \mathit{match}_{\text{next}}\ v, \\ \qquad\quad \mathit{snd}\ k) \\ \qquad \mid \_ \to \mathit{match}_{\text{next}}\ v, \\ \quad) \\ \quad (\mathit{snd}\ k)\ )\cdots) \end{array}$$
- $[\![\texttt{raise}]\!] := \texttt{fun}\ e \to \texttt{fun}\ k \to (\mathit{snd}\ k)\ e$
- $[\![\texttt{let rec}\ f\ \texttt{=}\ e\ \texttt{in}\ e']\!] := \texttt{fun}\ k \to [\![(\!|e_1|\!)^f]\!]\ (\texttt{fun}\ u \to \texttt{let rec}\ f\ x\ \texttt{=}\ u\ (f, x)\ \texttt{in}\ [\![e_2]\!]\ k, \mathit{snd}\ k)$

On définit
- $\mathit{fst} := \texttt{fun}\ (x, y) \to x$
- $\mathit{snd} := \texttt{fun}\ (x, y) \to y$

et où $(\!|e|\!)^f$ est une fonction partielle définie par induction (*il y a 17 cas*)
- $(\!|n|\!)^f$ n'est pas défini
- $(\!|x|\!)^f$ n'est pas défini
- $(\!|b|\!)^f$ n'est pas défini
- $(\!|\texttt{()}|\!)^f$ n'est pas défini
- $(\!|e_1\ e_2|\!)^f$ n'est pas défini
- $(\!|e_1 \circledast e_2|\!)^f$ n'est pas défini
- $(\!|\circledast\ e|\!)^f$ n'est pas défini
- $(\!|\texttt{for}\ i\ \texttt{=}\ e_1\ \texttt{to}\ e_2\ \texttt{do}\ e_3\ \texttt{done}|\!)^f$ n'est pas défini
- $(\!|\texttt{for}\ i\ \texttt{=}\ e_1\ \texttt{downto}\ e_2\ \texttt{do}\ e_3\ \texttt{done}|\!)^f$ n'est pas défini
- $(\!|\texttt{while}\ b\ \texttt{do}\ e\ \texttt{done}|\!)^f$ n'est pas défini
- $(\!|\mathrm{C}(e_1, \dots, e_n)|\!)^f$ n'est pas défini
- $(\!|e_1\ \texttt{;}\ e_2|\!)^f := e_1\ \texttt{;}\ (\!|e_2|\!)^f$
- $(\!|\texttt{if}\ e\ \texttt{then}\ e_1\ \texttt{else}\ e_2|\!)^f := \texttt{if}\ e\ \texttt{then}\ (\!|e_1|\!)^f\ \texttt{else}\ (\!|e_2|\!)^f$
- $(\!|\texttt{match}\ e\ \texttt{with}\ p_1\ \texttt{when}\ e_1' \to e_1 \mid \cdots \mid p_n\ \texttt{when}\ e_n' \to e_n|\!)^f := \texttt{match}\ e\ \texttt{with}\ p_1\ \texttt{when}\ e_1' \to (\!|e_1|\!)^f \mid \cdots \mid p_n\ \texttt{when}\ e_n' \to (\!|e_n|\!)^f$
- $(\!|\texttt{try}\ e\ \texttt{with}\ p_1\ \texttt{when}\ e_1' \to e_1 \mid \cdots \mid p_n\ \texttt{when}\ e_n' \to e_n|\!)^f := \texttt{try}\ (\!|e|\!)^f\ \texttt{with}\ p_1\ \texttt{when}\ e_1' \to (\!|e_1|\!)^f \mid \cdots \mid p_n\ \texttt{when}\ e_n' \to (\!|e_n|\!)^f$
- $(\!|\texttt{let rec}\ f\ \texttt{=}\ e\ \texttt{in}\ e'|\!)^f := \texttt{let rec}\ f\ \texttt{=}\ e\ \texttt{in}\ (\!|e'|\!)^f$
- $(\!|\texttt{fun}\ x \to e|\!)^f := \texttt{fun}\ (f, x) \to e$

L'unique cas de base dans la définition de $(\!|e|\!)^f$ est une fonction.